Similar Solutions of Boundary-Layer Equations for Power-Law Fluids

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The authors have recently obtained (*1*) the condition under which the boundary-layer equations for two-dimensional flows of power-law fluids (*2*) admit of similar solutions. Schowalter (*3*) used a somewhat different method for the three-dimensional case of the flow past a flat plate where the potential velocity vector is not perpendicular to the leading edge of the plate.

The object of the present note is to point out a small error in the mathematical development of (*3*) and to give the necessary corrections.

We use the notation of (3). The corrected second term on the L. H. S. of Equations (14), (15), and (16) should be

$$\frac{g^{n+1} W^o}{(U^o)^n} \frac{\partial U^o}{\partial z^o} [F'G' - 1] \quad (1)$$

$$\frac{W^o}{(U^o)^{n-1}} \frac{\partial \ln U^o}{\partial x^o} [F'G' - 1] \quad (2)$$

and

$$\frac{U^o}{(W^o)^{n-1}} \frac{\partial \ln W^o}{\partial x^o} [F'G' - 1] \quad (3)$$

The modified Equation (17) is

$$(U^o)^{1-n} \frac{\partial U^o}{\partial x^o} = a_1 W^o (U^o)^{1-n} \frac{\partial \ln U^o}{\partial z^o} - a_2 (U^o)^{1-n} \frac{\partial W^o}{\partial z^o} =$$

$$a_3 (U^o)^{2-n} \frac{\partial \ln g}{\partial x^o} = a_4 (U^o)^{1-n}$$

$$W^o \frac{\partial \ln g}{\partial z^o} = \frac{a_5}{g^{n+1}} = a_6 (W^o)^{1-n}$$

$$\frac{\partial W^o}{\partial z^o} = a_7 U^o (W^o)^{1-n} \frac{\partial \ln W^o}{\partial x^o} =$$

$$a_8 (W^o)^{1-n} \frac{\partial U^o}{\partial x^o} = a_9 (W^o)^{2-n} \frac{\partial \ln g}{\partial z^o}$$

$$= a_{10} (W^o)^{1-n} U^o \frac{\partial \ln g}{\partial x^o} \quad (4)$$

From Equation (4) we get

$$W^o = k\, U^o \quad (5)$$

and

$$(U^o)^{1-n} \frac{\partial U^o}{\partial x^o} = a_1 k\, (U^o)^{1-n} \frac{\partial U^o}{\partial z^o} =$$

$$a_2 k\, (U^o)^{1-n} \frac{\partial U^o}{\partial z^o} = a_3 \frac{(U^o)^{2-n}}{g} \frac{\partial g}{\partial x^o}$$

$$= a_4 k\, (U^o)^{2-n} \frac{1}{g} \frac{\partial g}{\partial z^o} = \frac{a_5}{g^{n+1}} =$$

$$a_6 (kU^o)^{1-n} \frac{\partial U^o}{\partial z^o} = a_7 (kU^o)^{1-n}$$

$$\frac{\partial U^o}{\partial x^o} = a_8 (kU^o)^{1-n} \frac{\partial U^o}{\partial x^o} =$$

$$a_9 (kU^o)^{2-n} \frac{1}{g} \cdot \frac{\partial g}{\partial z^o} =$$

$$a_{10} k^{1-n} (U^o)^{2-n} \cdot \frac{1}{g} \frac{\partial g}{\partial x^o} \quad (6)$$

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Impact Tube Size in Fluid Velocity Measurement

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The use of impact tubes for fluid velocity measurement requires that four corrections on the raw data be made. The first correction is for velocity gradient (*1*). When the velocity profile is steep, as it is near a wall, it is necessary to take the size of the impact opening into account. The second adjustment is for fluid viscosity (*2*) and is particularly important for highly viscous fluids. Both of these corrections are significant near the wall and become negligible in the turbulent core. The third correction is due to turbulent fluctuations (*4*) and is negligible near the wall, but must be taken into account in the turbulent core region.

The fourth correction is for pressure variation over the impact area and is important in all flow regions. It is this correction to which we wish to confine our attention at the present time. The correction arises because of the nature of the flow around the entire impact tube, and the adjustment is greater for straight impact tubes than for Pitot or bent impact tubes.

In the past, investigators (*3 to 8*) using impact tubes for velocity measurements have reported that their data fall below that obtained by other techniques. Rosler and Bankoff (*4*) have concluded that velocity measurements for submerged water jets using impact tubes give results which are about 12% below those obtained using a hot-wire anemometer. Rothfus and co-workers (*5, 6*), working on flow in conduits, have used several sizes of impact tubes and extrapolated to zero diameter. This method accounts for all but the viscous and turbulent fluctuation corrections.

Consider a fluid flowing steadily past a circular cylinder with its longitudinal axis at right angles to the direction of flow. The pressure distribution on the forward surface of the cylinder is given

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Longitudinal dispersion of thermal energy through porous media with a flowing fluid, Green, D. W., R. H. Perry, and R. E. Babcock, **A.I.Ch.E. Journal, 10,** No. 5, p. 645 (September, 1964).

Key Words: Temperature-1, Energy Balance-2, Porous Media-5, Packed Beds-5, Holding Time-6, Heat Transfer Coefficient-6, Conduction-6, Peclet Number-6, Mixing-6, Velocity Profiles-6, Channeling-6, Convection-6, Conductivity-7, Dispersion-7, Diffusivity-7, Heat Transfer-8, Mass Transfer-8, Fluid Flow-8, Digital Computer-10, Numerical Solution-10, Approximate Solution-10, Convective-, Eddy-, Longitudinal-, Gaussian-, Error Function-, IBM-.

Abstract: The rate of heat transfer in porous media with a flowing fluid present is examined theoretically and experimentally. The mechnisms considered are bulk movement of the fluid, conduction in the solid and fluid phases, convective transfer of heat between the phases, and convective eddy mixing or dispersion of the fluid phase. Tht describing differential equations are solved numerically with a digital computer. An approximate solution is presented which is based on the diffusivity-equation solution and the additivity of the individual dispersion mechanisms. Experiments are conducted in glass bead-liquid systems at low liquid velocities. Overall effective thermal conductivities (dispersion coefficients) are calculated from the data. The applicability of the mathematical model is verified.

Kinetics of hydrogen reduction of ferrous oxide in a batch fluidized bed: effects of mass velocity and pressure, Ahner, W. D., and Jerome Feinman, **A.I.Ch.E. Journal, 10,** No. 5, p. 652 (September, 1964).

Key Words: A. Reduction-8, Kinetics-8, Ferrous Oxide-1, Oxides (Inorganic)-1, Hydrogen-1, Iron-2, Water Vapor-2, Fluidized Bed-10, Reactor-10, Pressure-6, Gas Mass Velocity-6, Temperature-6, Reduction Path-7. B. Design-8, Fluidized-Reduction Processes-10, Batch-, Continuous-.

Abstract: The kinetics of reduction of ferrous oxide with hydrogen were studied in an isothermal batch fluidized reactor. The effects of mass velocity and pressure on the reduction path were investigated at 1,000° and 1,300°F. The data are correlated by means of instantaneous oxygen balances. Application of the results to the design of commercial fluidized-reduction processes is discussed qualitatively.

An experimental study of surface cooling by bubbles during nucleate boiling of water, Rogers, Thomas F., and Russell B. Mesler, **A.I.Ch.E. Journal, 10,** No. 5, p. 656 (September, 1964).

Key Words: Nucleate Boiling-1, Surface Cooling-2, 7, Water-5, Bubble Formation-6, Heat Transfer-8, High-Speed Photography-10, Fast Response Surface Thermocouples-10.

Abstract: To test a hypothesis of Moore and Mesler concerning surface cooling during bubble growth in nucleate boiling of water a technique was developed to photograph a growing bubble as the surface temperature was being measured with a fast response thermocouple. Surface temperature was found to cool during bubble formation but then rise during bubble departure in a manner consistent with the hypothesis.

Vapor-liquid equilibrium determination by a new apparatus, Yerazunis, Stephen, J. D. Plowright, and F. M. Smola, **A.I.Ch.E. Journal, 10,** No. 5, p. 660 (September, 1964).

Key Words: Liquid-1, Toluene-1, n-Heptane-1, n-Butyl-Alcohol-1, Benzene-1, Vapor-2, Vaporization Rate-6, Recirculation Time-6, Equilibrium-7, Thermodynamic Consistency-8, Equilibrium Still-10.

Abstract: A new equilibrium still employing vapor and liquid recirculation and distinguished by a final equilibration chamber through which the vapor and liquid streams pass in concurrent flow prior to separation has been evaluated. Equilibrium measurements for the system n-hexane-toluene at 1 atm. are in excellent agreement with published results. The experimental results are of high internal consistency, ± 0.001 in mole fraction and ± 0.05°C., and are thermodynamically consistent. Vapor-liquid equilibrium data for the system n-butyl alcohol-benzene are also presented. Ease of operation and shortness of equilibration time are attractive features of the apparatus.

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by (9)

$$\Delta P = \frac{\rho U^2}{2g_c} (1 - 4 \sin^2\theta \qquad (1)$$

If the impact area consists of a circle of radius r on a cylinder of radius R, and the area is situated so that the front stagnation line passes through its center, the average impact pressure exerted on this area is

$$\Delta P_{av} = \frac{\int_0^r \Delta P\, x dy}{\int_0^r x dy} \qquad (2)$$

where $\sin\,\theta = y/R$ and $x^2 + y^2 = r^2$. When Equations (1) and (2) are combined and integrated

$$\Delta P_{av} = \frac{\rho U^2}{2g_c}\left(1 - \frac{r^2}{R^2}\right) \qquad (3)$$

This relationship gives the average pressure exerted over the impact area for a straight impact tube. Equation (1) is based on the flow of nonviscous fluids, but also may be applied to viscous fluids (9) up to a value of θ = 60 deg. which covers the design range of most impact tubes.

In conjunction with some mass transfer studies, the authors have used an impact tube to measure velocities for fully developed turbulent flow of air in a pipe. For the fairly conventional dimensions of R = 0.0178 and r = 0.0080 in. Equation (3) becomes

$$U = 1.122\ (2g_c\, \Delta P_{av}/\rho)^{1/2} \qquad (4)$$

At a Reynolds number of 14,000 the raw data for the turbulent core region had a scatter of approximately 2% which was almost entirely due to the limitations of the micromanometer. If the conventional impact tube equation, $r = 0$ in Equation (3), was used, the results were consistently about 10% below the correlation proposed by Deissler (*10*). When Equation (3) was used the data fell consistently about 3% above the Deissler correlation.

The data were brought into agreement with the Deissler relation by using Equation (3) with the turbulent fluctuation correction. This was done by resolving the instantaneous velocity into a time-average component and a fluctuating component and time averaging the result. Neglecting static pressure fluctuations, lateral velocity fluctuations, and the size of the impact tube

$$\Delta P = \frac{\rho}{2g_c} (U^2 + U'^2) \qquad (5)$$

Following the analysis for Equation (3) yields

$$U = \left[\frac{2g_c \, \Delta P_{av}}{\rho\left(1 - \frac{r^2}{R^2}\right)} - \overline{U'^2} \right]^{1/2} \quad (6)$$

Values of $\overline{U'^2}$ were taken from hot-wire anemometer data *(10, 11)*.

Equation (3) applies directly in the center portion of the pipe for laminar flow of low-viscosity fluids. For turbulent flow the corrections for pressure variation over the impact area and for turbulent fluctuations have a compensating effect as illustrated by Equation (6).

ACKNOWLEDGMENT

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NOTATION

g_c = gravitation constant
ΔP = difference between impact pressure at any point on the impact area and the static pressure
ΔP_{ave} = average pressure difference over the impact area
r = radius of the impact area
R = radius of the impact tube
ρ = density of the fluid
θ = angle between a point on the impact area and the front stagnation line
U = velocity of the fluid at the impact point
x = axial distance on the cylinder from the center of the impact area
y = radial distance on the cylinder from the center of the impact area

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Fluidised Particles, J. F. Davidson and D. Harrison, Cambridge University Press, New York (1963). 155 pages. $6.50.

This book is an interesting attempt to present a mathematical picture of the complicated phenomena of fluidization through an analytical treatment of the mechanism of fluidization, including incipient fluidization velocity, the formation of bubbles, the rise and coalescence of bubbles, the exchange between phases, and the stability of bubbles.

The analysis is heavily based on experimental work from the authors' laboratory, and these observations and measurements differ in many cases from those of other investigators. This lessens the value of the mathematical models. For example, the book states in several places that in aggregate fluidization "with still greater flows, the bubbles grow and appear more frequently until their frontal diameters are equal to the diameter of the containing apparatus," and again, "for a gas fluidised bed of one to two inch diameter, the bubble size becomes equal to the bed diameter when the height is more than one to two feet." This is not in agreement with the observations of other investigators who have found that with many aggregate systems the solid is blown out of the bed long before the bubble size is equal to the column diameter. These statements also appear to contradict later discussions in the book. Perhaps this is due to a different interpretation of aggregate fluidization.

The mathematical models used are, in general, very idealized and appear to neglect many important factors such as the presence of solid particles within the bubbles and the effects of radial velocity gradients in the column which other data indicate are of prime importance in the scale-up of fluidized-solid beds. The section on the velocity of bubbles neglects other studies which indicate that much higher velocities than the superficial velocity plus the idealized velocity of bubbles in a liquid are present. The use of the "stationary bubble-moving fluid" analog is not a good approximation of actual conditions.

The section on the fluidized bed as a catalytical reactor does not give an adequate analysis of the advantages and disadvantages of these units as compared to other types of reactors or of the real problems of scaling-up from laboratory units. This section also ne-

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